Distributed Scheduling and Cooperative Control for Charging of Electric Vehicles at Highway Service Stations

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***Abstract*— The increasing number of electric vehicles (EVs) on highways calls for the installment of adequate charging infrastructure. Since charging infrastructure has limited capacity, EVs need to wait at a charging station to get charged, and their waiting times may differ significantly from one location to another. This paper aims at developing a strategy to coordinate the queues among the charging stations, with only local information about traffic flows and the status of EV charging stations along a bidirectional highway, so that excessively long waiting times can be avoided. Specifically, a distributed algorithm is presented to schedule EV flows into neighboring charging stations, so that EVs are all appropriately served along the highway and that all the charging resources are uniformly utilized. In addition, a distributed decision making policy is developed to influence the aggregate number of EVs entering any given service station, so that each EV makes an appropriate decision (i.e., whether or not it should enter the next charging station) by contributing positively to meeting the desired queue length at service stations and by considering its own battery constraint. Performance improvement of the proposed strategy is illustrated via one of the highways in the United States, namely the Florida Turnpike.**

***Index Terms*— Electric vehicles' charging, transportation networks, distributed consensus algorithm, cooperative control, dynamical systems.**

I. Introduction

It IS expected that by 2020 there will be not less than 3.1 million of electric vehicles (EVs) in use in the United States [2]. The adoption of a large number of EVs can bring many advantages in comparison to internal combustion engine vehicles, including higher efficiency, lower carbon emission and less pollution [3], [4]. However, EVs have a more limited driving range (from 38 to 270 miles [5]) and, for long distance travels, they need to be charged periodically along the way. Therefore, the combination of the limited driving range, the long charging times (30 mins to 8 hours [2]), and the customer's satisfaction level when they wait for their EVs to get charged may have a direct impact on decisions by consumers to adopt the use of EVs in the future.

Numerous factors including installment of new charging infrastructure, advances in battery and charging technology, and development of smart (coordinated) charging strategy play an important role in addressing the above challenges. This paper focuses on the EVs' charging scheduling problem, a current topic of research, whose goal is to optimally utilize the existing charging network or infrastructure by taking advantage of information and communication technologies (ICT). In particular, we study the scenario that along a bidirectional highway there are a number of service stations equipped with EV chargers. The chargers are located at service stations (typically close to a set of entrances and exits) where EV drivers would choose to charge their vehicles. It is obvious that, while battery charging time depends only upon the charger technology, waiting times of EVs at service stations are highly dependent upon traffic flows and may become unacceptably long with a local increase of EVs entering a given service station. Without appropriate scheduling and coordination, utilization of the charging stations on the highway may become very unbalanced since EV drivers do not have adequate information to decide where and when to charge their vehicles (e.g., each individual EV chooses randomly where to charge their batteries). This calls for both a scheduling algorithm of directing EV flows into service stations and a distributed decision model to facilitate individual EV drivers to make better charging decisions.

Literature review: Apart from studies on charging station infrastructure planning (e.g., [6]–[8]), the EVs' charging scheduling problem has also received considerable attention in the literature, see e.g. [9]–[26].

Qin and Zhang [22] propose a (centralized) scheduling algorithm based on Dijkstras algorithm and by combining both the transport and power grid information to find optimal path for the EVs which improves the performance of the grid and transport systems, namely relieving traffic congestion, reducing power loss and optimizing load curve. Similarly, a hierarchical game approach is presented in [25] for navigation of EVs' charging by taking into account the impact from both the transportation and power systems and aimed at improving the economic benefits of the charging stations and the reliability of the power grid.

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Apart from the above results which consider both the transportation and power systems, much of the literature on EVs' charging scheduling has focussed solely on the transportation system for either urban or highway environments with the goal of balancing the demand across the charging network, reducing the queue length, or minimizing total travel time and waiting time at the charging stations. For example, the work in [20] proposes a dynamic charging assignment method for the EVs to optimize the charging station utilization and thus minimize the EVs' waiting time. The algorithm is computed via Simulated Annealing algorithm by a centralized control platform. Li *et al.* [9] compare two charging station selection algorithms based on local information of an EV and global information obtained from a central server. It is shown that utilization of global information on the charging station workload helps improving the waiting time for the EVs. A routing problem is solved offline and in a centralized manner based on dynamic programming in [24] for EVs on a network of charging nodes whose goal is to minimize the total traveling and charging time. Coninx *et al.* [19] consider a scenario where EV drivers inquire the central controller for advice about specific charging station to choose and the relevant route to use. The drivers are to find the best tradeoff between the whole trip time, including waiting time at the charging station. The problem is solved in a centralized manner using the global information on the road traffic, location of charging stations and their occupancy. An algorithm of directing EV flows to charging stations is proposed in [12] to distribute the charging load and to minimize queuing time by having the EVs communicate with the transportation network.

Note that all the aforementioned results require global network information and centrally computing algorithms. Distributed EV charging and scheduling problem has recently been investigated to a limited extent, and a few results are now available. Bodet *et al.* [21] propose a distributed strategy based on charging station reservation for scheduling the EV's charging so that the average waiting time is minimized. First, it is proven under certain assumptions (e.g. each station has equal charging capacity) that the waiting time is minimized if utilizations of all stations are equal. Based on this insight, the authors then propose a distributed method for locally balancing distribution of charging demands in order to approximate the system-wide distribution. A decentralized policy is designed in [23] to assign EVs to a network of charging stations with the goal of minimizing the queueing time. However, no precise analytic guarantees on load balancing is provided. Closer to our proposed method, Tan and Wang [26] propose a distributed scheduling approach based on A* algorithm and that makes use of charging station reservation system aimed at minimizing the total travel time for each EV.

It is worth noting that, in all the results of distributed scheduling, there is no rigorous analysis or proof, and validation is done by simulation alone. In addition, the dynamics/changes in the transportation system such as time varying traffic flow are not explicitly considered in the design of the algorithms. Recent survey of algorithms for EV's charging can be found in [27] and [28].

Statement of contributions: We propose an analytically-proven algorithm to distributively schedule EV's charging and optimize the utilization of charging resources along a highway.

Our first contribution is the development of dynamical model for average flow of EVs along the highway coupled with queues at the service stations. The dynamical model is used to design and investigate stability of the algorithm which adapts to the changes of communication network (in connectivity and delay) and transportation network (in flows and charging equipment availability). In contrast to the mathematical model of EV's charging demand based on fluid dynamic traffic model (represented by partial differential equation) in [29], our model is an aggregated average model in the discrete time domain and its simplicity/flexibility facilitates the analysis and design of the scheduling algorithm with performance guarantee.

The second contribution is the development of a *distributed* scheduling algorithm and a cooperative control algorithm for EVs' charging. In contrast to *centralized* algorithm where all data for the optimization or control is collected at a central aggregator which may require high-bandwidth communication and thus pose a potential bottleneck and delay, distributed algorithm is highly desirable due to its scalability (which reduces the computational and communication costs) with respect to the size of the EVs and charging network since it only requires local knowledge about the system. In a centralized system, the central aggregator is also vulnerable to a single point-of-failure which may yield the collapse of the whole system. Moreover, since for the case of centralized algorithm all data including the one containing the privacy of the EV drivers is collected at a central aggregator, the risk of exposing the data to cyber-attacks is also increased. On the other hand, distributed algorithm is robust to information intermittency in both communication networks and transportation network. For example, when a communication link between the service stations is broken, the information can still be transmitted using an alternative communication path which makes the entire system remain functional.

The distributed scheduling algorithm is inspired by the consensus algorithm and relies solely on information of local traffic flows at the neighboring service stations. Its objective is to schedule and direct the percentage of the EVs that enter each station. Note that recently the consensus algorithm has been applied to intelligent transportation systems such as for controlling vehicle platoons for highway safety [30] or for controlling traffic density in a highway network [31].

Based on the optimized number of EVs that should enter each service station, a distributed algorithm of cooperative control is used as the decision making algorithm by taking advantage of local vehicle-to-infrastructure and vehicle-to-vehicle communications. Specifically, this algorithm is designed for each EV to determine whether or not to pass a specific service station by taking into account its own battery charge constraints while meeting the desired flow level from the scheduling algorithm. The combination of the two algorithms ensures a uniform utilization of the available charging facility along the highway and in turn minimizes the aggregate waiting time of all the EVs that need to be charged as shown in [21].

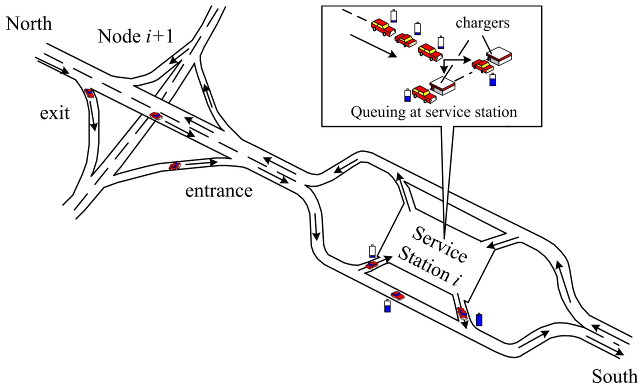


Fig. 1. A bidirectional highway segment with exit/entrance and service station.

Note that our optimization objective is global in the sense that it involves (explicitly) all charging facilities on the highway, in contrast to the work in [26] which considers the (local) minimization of the performance of individual EV. Finally, in comparison to the preliminary version of our work on this problem [1], we propose in this paper a better distributed scheduling algorithm together with an improved decision making algorithm that preserves data privacy of each EV, and we also provide extensive simulations using the real traffic flow data from the Florida turnpike.

Organization: The rest of the paper is organized into 4 sections. In section II, discrete-time dynamic models are presented to describe the average flow of electric vehicles passing through the entrances/exits and entering the service stations, the proposed scheduling and control problem of electric vehicle charging is defined, and the objectives of the proposed algorithms are prescribed. In section III, the proposed consensus algorithm of distributed scheduling is presented for the transportation network, and its properties are rigorously analyzed. In section IV, the decision making algorithm based on cooperative control is proposed for individual drivers and studied. In section V, the Florida turnpike is used as the case study, and its traffic flow data are used to evaluate efficacy of the proposed algorithms. In section VI, conclusions are drawn.

II. Modeling & Problem Statement

Highways, such as the Florida Turnpike, consist of a fixed number of entrances/exits, service stations typically locate at or close to a subset of these entrances/exits, and these stations are upgraded to have EV chargers, which is illustrated by figure 1.

In what follows, we first model the average flow of EVs entering/passing through entrances/exits. Then, we choose to use the M/M/c queueing model to describe the number of EVs entering and waiting to be charged at service stations. These models contain the decision variables for which optimization and control problems can be formulated and solved. The scheduling problem aims at uniform utilization of the charging infrastructure within the transportation network, the problem of intelligent decision making by individual drivers calls for coordination with the network and among drivers, and both the problems need to be solved without requiring global information. For the sake of readability, the notations used in this paper are summarized in Table I.

TABLE I
A List of Notations

Notation	Description
N	Number of nodes representing extrances/exits or service stations
$\alpha_i(k)$	Average EV flow approaching the ith node at time k
$\gamma_i(k)$	Net flow increase at node i at time k
$y_{i-1}(\cdot)$	Average EV flow coming from node $(i-1)$ to node i
$d_{i-1,i}$	Travel time from node $(i-1)$ to node i
$g_i(k)$	Flow coming out from service station i
$f_i(k)$	Average flow entering service station i
$x_i(k)$	Number of EVs at service station i at time k
$p_i(k)$	Percentage of EVs approaching station i which choose to enter the station
c_i	Number of EV chargers at station i
μ_i	Charging times (service rate) at station i
$e^-_{v,i+1}$	State of charge of the vth vehicle when approaching node $(i+1)$
$e^+_{v,i}$	State of charge of the vth vehicle when leaving node i
r^-_v	Battery usage rate of the vth vehicle
$u_{v,i}$	Binary decision variable on whether to charge at station i for the vth vehicle
$e_{v,max}$	Maximum battery capacity of the vth vehicle
$\beta_i(k)$	Average number of charging required per vehicle for EVs passing through node i at time k
$\eta_i(k)$	Estimate of total charging needs by node i
$\eta_0(k)$	Desired utilization value for all service stations

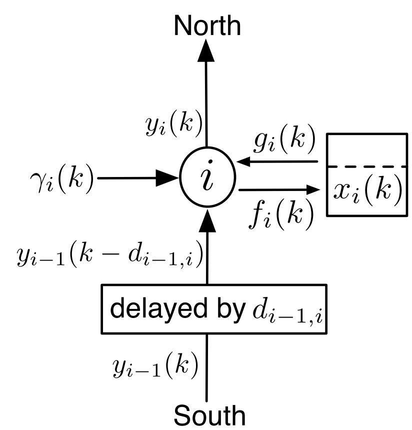


Fig. 2. A model of average EV flow at the ith extrance/exit.

A. A Model for Average EV Flow

Consider an one-direction traffic flow along the highway segment, for example the northbound flow, and assume that there are N nodes (which include both extrances and exits) and they are numbered in an ascending order from south to north, as shown in figure 1. Then, the following discrete-time model in figure 2 can be used to quantify the average traffic flow. That is, the average northbound vehicle flow $\alpha_i(k)$ that approaches the ith node is defined as

$$\alpha_i(k) = \begin{cases} \gamma_i(k), & \text{when } i = 1 \\ \gamma_i(k) + y_{i-1}(k - d_{i-1,i}), & \text{if } i \neq 1 \end{cases}$$

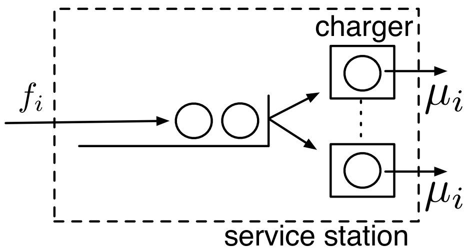


Fig. 3. A queueing model for service station at node i.

where k is the unit time increment, γ_i is the net northbound flow increase (due to vehicles entering node i from local roadways through entrance/exit i), $y_{i-1}(k - d_{i-1,i})$ is the average EV flow coming from node $(i-1)$ to node i, and $d_{i-1,i}$ denotes the travel time (of unit time increment) needed from one node to the next. At time k, the northbound continuing flow from the ith node can be calculated using

$$y_i(k) = \alpha_i(k) + g_i(k) - f_i(k), \quad (1)$$

where $g_i(k)$ is the northbound flow coming out from service station i, and $f_i(k)$ is the average flow entering the ith service station. Should node i have no service station, we have $g_i(k) = f_i(k) = 0$.

B. A Queueing Model for Service Stations

Let the number of EVs at service station i at time k be denoted by $x_i(k) \geq 0$. The EV flow from the highway interacts with queue state $x_i(k)$ at the ith service station according to the following dynamics:

$$\begin{aligned} x_i(k+1) &= x_i(k) + f_i(k) - g_i(k), \\ f_i(k) &= p_i(k)\alpha_i(k), \end{aligned} \quad (2)$$

where $p_i(k) \in [0, 1]$ is the percentage (portion) of EVs that approach the ith node and choose to enter its service station i.

Let c_i be the number of EV chargers at service station i which always serve from the front of its queue. As illustrated by figure 3, an EV enters a service station by getting into the charging queue there until one of charging stations becomes open for it to use (i.e., first-come-first-served rule). Moreover, we assume that the capacity for the EVs to queue is sufficiently large.[1] We utilize the stochastic model to analyze the queue since the EVs arrival time and the service time at the station are not known, i.e., random in general. Specifically, we model the relationship between $f_i(k)$, $\mu_i(k)$ and $x_i(k)$ as an M/M/c_i queue, a standardized model for birth-death processes depicted in figure 4. To be more precise, the M/M/c_i queue as a stochastic process has its states defined by the set $\aleph = \{0, 1, 2, 3, \cdots\}$. In other words, the queue is the Markov process denoted by $\{x_i(k) = l : l \in \aleph\}$. In this queue model, the mean arrival rate of the EVs at service station i, denoted by $f_i(k)$, is modeled as a Poisson process. The assumption on Poisson arrival process for the EVs is appropriate (and has been verified by experiments in [32]) since in general the total number of vehicles on the highway is very large,

[1]Otherwise, another model could be used and that model can include the impact on continuing traffic flow.

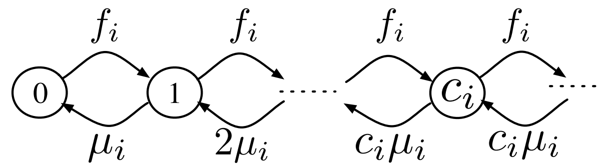


Fig. 4. Representation of a birth-death process for M/M/c_i queue model.

a single EV uses a small percentage of highway and each EV makes independent decision to enter the highway/service station [33]. Moreover, the charging times (i.e., service rate) is given by μ_i and has an exponential distribution, which is confirmed for constant charging power by the commute distance distribution in [34]. Note that the M/M/c_i queue model has also been widely used in the EV's charging related literature for purpose of analysis, see e.g. [12], [21], [25], [29], [35]–[37]. The boundedness of the queue is guaranteed if the following condition is satisfied at the steady state:

$$f_i < c_i\mu_i \quad (3)$$

namely, the steady-flow number of EVs entering charging station i should be less than capacity of the station.

C. A Simple Energy Model of EVs

In general, the energy consumption of EVs depend on many factors such as driving cycles [33]. However, for the sake of simplicity in this paper we model the electricity consumption of the υth EV as

$$e^-_{\upsilon,i+1} = e^+_{\upsilon,i} - d_{i,i+1} r^-_{\upsilon}, \quad (4)$$

where $e^-_{\upsilon,i+1}$ is the state of charge of the υth vehicle as it approaches node $(i+1)$, $e^+_{\upsilon,i}$ is the state of charge as it leaves node i, and r^-_{υ} is the battery usage rate of the υ-th vehicle. Let $u_{\upsilon,i} \in \{0, 1\}$ be the decision variable for individual EV and is defined as

$$u_{\upsilon,i} = \begin{cases} 1 & \text{if vehicle } \upsilon \text{ choose to charge at} \\ & \quad \text{service station } i \\ 0 & \text{otherwise.} \end{cases} \quad (5)$$

When the υth EV departs service station i, its state of charge status becomes

$$e^+_{\upsilon,i} = u_{\upsilon,i} e_{\upsilon,max} + (1 - u_{\upsilon,i}) e^-_{\upsilon,i}, \quad (6)$$

where $e_{\upsilon,max}$ denotes the maximum capacity of its battery. Models (5) and (6) assume that an EV will leave with a full charge after entering a service station. Other models can be accommodated similarly so that partial charging or earlier departure can also be considered.

D. Distributed Scheduling and Cooperative Control

The objective of this paper is two-fold. The first goal is to design decision variables $p_i(k)$ in (2) for each service station so that the total charging demands are allocated equally across the network. In other words, we aim to optimize operation of highway network, namely all service stations on the highway are uniformly utilized. Let $\frac{x_i(k)}{c_i\mu_i}$ denote the utilization of

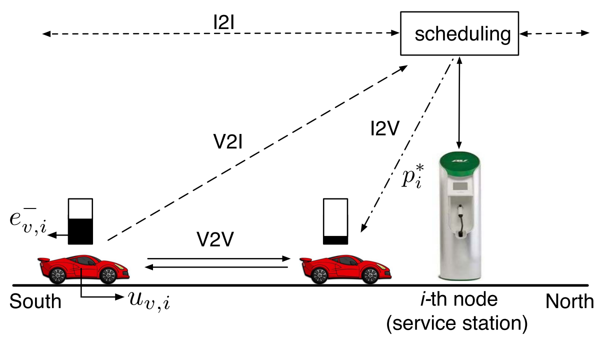


Fig. 5. Distributed scheduling and cooperative control of EVs using infrastructure-to-infrastructure (I2I), vehicle-to-vehicle (V2V), vehicle-to-infrastructure (V2I) and infrastructure-to-vehicle (I2V) communication.

service station i. The problem can be mathematically stated as to solve the following optimization problem:

$$\min_{p_i(k)} \underbrace{\left[\max_i \left(\frac{x_i(k)}{c_i\mu_i}\right) - \min_j \left(\frac{x_j(k)}{c_j\mu_j}\right)\right]}_{=J} \tag{7}$$

distributively using only the (local) information of $y_{i-1}(k - d_{i-1,i})$, $x_{i-1}(k-1)$, $\gamma_i(k)$, $x_i(k-1)$, and $x_{i+1}(k-1)$ from the neighboring service stations.

The second goal is to design a cooperative decision making algorithm for individual EV, i.e., $u_{v,i}(k)$ in (6), to choose a specific service station for charging by attempting to meet the optimally scheduled flow level in (7) while taking into consideration its own state of charge, which is illustrated by figure 5. In addition, the policy should also guarantee the EVs with low State-of-Charge (SoC) to receive first priority to be charged at the nearest service station. In order to formulate the problem, let $\mathcal{N}_i(k)$ be the set of EVs approaching the ith service station and without loss of any generality, we assume the size of the set $|\mathcal{N}_i(k)|$=$\alpha_i(k)$. Furthermore, let the solution to (7) be given by $p_i^*(k)$. When approaching service station i, the EVs interact with neighboring EVs from the set $\mathcal{N}_i$ and receive the information of $p_i^*(k)$ from the service station by means of wireless communication as shown in figure 5. The EVs negotiate with the neighboring EVs based on its battery status to choose the best service station for it to get charged. The problem can then be mathematically stated as to solve the following optimization problem:

$$\begin{aligned} &\min_{u_{v,i}(k)} \left[p_i^*(k) - \frac{\sum_v u_{v,i}(k)}{\alpha_i(k)}\right]^2 \\ &\text{subject to } \; e_{v,i}^+ \geq d_{i,i+1} r_v^- \text{ if } i \in \mathcal{N}_i(k), \\ &\qquad\qquad u_{v,i}(k) \in \{0,1\}. \end{aligned} \tag{8}$$

The first constraint ensures that an EV will be charged at service station i whenever it does not have sufficient energy to reach the next service station.

III. Consensus-Based Distributed Scheduling Algorithm

In this section, we develop a scheduling algorithm to solve the distributed optimization problem (7). To this end, note that since $x_i(k) \geq 0$, we have $J \geq 0$. The minimum of (7) is then given by $J_{min} = \min J = 0$ obtained when

$$\frac{x_1(k)}{c_1\mu_1} = \cdots = \frac{x_N(k)}{c_N\mu_N}.$$

In other words, the cost function (7) is minimized when the utilization $\frac{x_i}{c_i\mu_i}$ reach a *consensus* for all service stations. In fact, the cost function J in (7) also serves as a Lyapunov function for the consensus protocol which will be presented in section III-C [38]. Therefore, the optimization problem (7) can be reformulated as to find $p_i^*(k)$ such that for all service stations i,

$$p_i(k) = p_i^*(k) \Longrightarrow \frac{x_i(k)}{c_i\mu_i} \to \eta_0(k). \tag{9}$$

In the remainder of this section, using the steady state solution of M/M/c_i queue and for flow dynamics given in (1) and (2), a consensus law is designed as the distributed solution to the scheduling problem that needs to be solved at the service stations. Moreover, we show convergence to a consensus followed by stability analysis of its value.

A. Approximation of EV Flow Out Using Steady State Solution of M/M/c_i Queue

Before proceeding, we first need to derive the analytical expression of EV flow out at service station i as a function of the state x_i. As described in section II-B, the charging service at service station i is modeled by the M/M/c_i queue with mean arrival rate $f_i(k)$ and service rate μ_i. It is known that the steady solution of $x_i(k)$ is equal to [39, p. 214]

$$\begin{aligned} x_i &= \frac{\rho_i^{c_i+1}}{c_i c_i!}\frac{1}{(1-\rho_i/c_i)^2}\varphi_{i,0} + \rho_i, \\ \varphi_{i,0} &= \left[\sum_{n=0}^{c_i-1}\frac{\rho_i^n}{n!} + \frac{\rho_i^{c_i}}{c_i!(1-\rho_i/c_i)}\right]^{-1}, \end{aligned} \tag{10}$$

where $\rho_i = f_i/\mu_i$ denotes the utilization of the chargers at service station i.

Next, at the steady state we have from (2) that the output flow from the service station is equal to $g_i = f_i$. Substituting this into (10), the input-output relationship between the dynamics (2) and the M/M/c_i queue can be written as

$$\begin{aligned} x_i &= \frac{g_i^{c_i+1}}{\mu_i^{c_i+1} c_i c_i!}\frac{1}{(1-g_i/\mu_i c_i)^2}\varphi_{i,0} + \frac{g_i}{\mu_i}, \\ \varphi_{i,0} &= \left[\sum_{n=0}^{c_i-1}\frac{g_i^n}{\mu_i^n n!} + \frac{g_i^{c_i}}{\mu_i^{c_i} c_i![1-g_i/(\mu_i c_i)]}\right]^{-1}. \end{aligned} \tag{11}$$

Note that an identical result to (11) is also obtained by applying the pointwise stationary approximation (PSA) method as presented in [40]. Obtaining the analytical expression for $g_i(x_i)$ from (11) is in general impossible. However, for the case of a single charger, i.e., $c_i = 1$, the steady state solution simply reduces to

$$x_i = \frac{g_i}{\mu_i - g_i} \quad \Longrightarrow \quad g_i = \mu_i \frac{x_i}{1+x_i}. \tag{12}$$

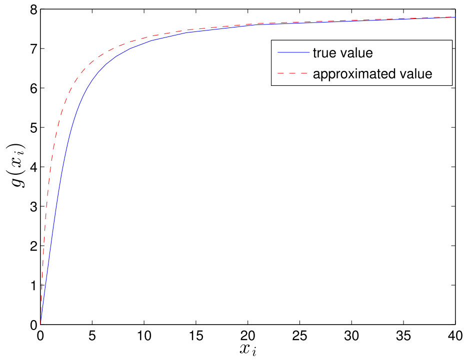


Fig. 6. Comparison between EV flow out function $g_i(x_i)$ in (11) and its approximation $\hat{g}_i(x_i)$ in (13): $\mu_i = 2$ and $c_i = 4$.

In order to derive the closed-form input-output solution of $g_i(x_i)$ (which is necessary to proceed with the design and analysis in the next subsection), in this paper we propose to decouple a single M/M/c_i queue into c_i M/M/1 queues. Using this idea and from (12), the input-output solution $g_i(x_i)$ to equation (11) can then be approximated by

$$\hat{g}_i(x_i) = c_i \mu_i \frac{x_i}{1 + x_i}. \tag{13}$$

We compare the solution (13) with the numerical solution to equation (11) in order to evaluate whether the proposed approximation is suitable, and the comparison is shown in figure 6. It is obvious from figure 6 that the approximation in (13) is suitable for analytical design. Note that due to the over-approximation, as shown in figure 6, it is possible that $\hat{g}_i(x_i) > x_i$ when $x_i < c_i \mu_i$ which can be interpreted as all EVs are leaving the station since the queue length is less than the charging station's capacity. In the simulation, a constraint $x_i \geq 0$ is added as mentioned in Section II-B so that the queue length will not be negative for this particular case.

B. Distributed Estimation Network of Total Charging Needs

Due to their constrained ranges, EVs need periodic charging during their long-haul travels. Let $\beta_i(k)$ denote the average number of charging required per vehicle for EVs passing through node i at time k. The value $\beta_i(k)$ is computed using the collected information at the ith node at time k including their battery status in (4) and their distances to destination. The total charging needs of the network is then given by $\sum_{j=1}^{N} \left[\beta_j(k)\gamma_j(k)\right]$. In the following we design an observer for each service station to estimate distributively the total charging needs of the network.

In order to gather locally information on charging needs, when an EV enters the highway at node i and at time $t = kT$, we assume that, with the help of *vehicle-to-infrastructure* (V2I) communication [41], it will register itself to be included in $\gamma_i(k)$, as depicted in figure 5. Each node then exchange information with their neighboring nodes by the means of *infrastructure-to-infrastructure* (I2I) communication and implement the following observer to distributively estimate the total charging need: for $t \in [kT,\ (k+1)T)$,

$$\begin{aligned}
\dot{\eta}_1 &= \zeta\left(-\eta_1 + \left[\frac{2\eta_1}{3} + \frac{\eta_2}{3}\right]\right)\\
\dot{\eta}_i &= \zeta\left(-\eta_i + \left[\frac{\eta_{i-1}}{3} + \frac{\eta_i}{3} + \frac{\eta_{i+1}}{3}\right]\right), \quad i = 2, \cdots, (N-1)\\
\dot{\eta}_N &= \zeta\left(-\eta_N + \left[\frac{2\eta_{N-1}}{3} + \frac{\eta_N}{3}\right]\right),
\end{aligned} \tag{14}$$

where ζ is the controller gain and

$$\eta_i(kT) = N\left[\beta_i(k)\gamma_i(k)\right].$$

Observer (14) can be compactly written in the matrix form as

$$\dot{\eta} = \zeta(-I + P)\eta, \tag{15}$$

where $\eta = [\eta_1 \ \dots \ \eta_N]^T$, I is the identity matrix and matrix P is given by

$$P = \begin{bmatrix}
\frac{2}{3} & \frac{1}{3} & 0 & 0 & \cdots & 0\\
\frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & \cdots & 0\\
 & \ddots & \ddots & \ddots & & \\
 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0\\
 & & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3}\\
 & & & 0 & \frac{1}{3} & \frac{2}{3}
\end{bmatrix}. \tag{16}$$

Observe that matrix P is row stochastic, column stochastic and irreducible. The convergence of the proposed distributed observer (14) is shown in the following lemma.

Lemma 1: By choosing the gain $\zeta \geq \frac{4}{\lambda_2 \delta T}$ where λ_2 is the second smallest eigenvalue of matrix $(I - P)$ and $0 < \delta < 1$, the total charging needs of the network can be estimated by distributed observer (14). Mathematically, we have

$$\eta_i(kT + \delta T) \to \sum_{j=1}^{N} \left[\beta_j(k)\gamma_j(k)\right].$$

Proof: It can be computed that the exponential convergence rate of observer (15) is given by $\zeta\lambda_2$ and its settling time δT is equal to $\delta T = 4(1/\zeta\lambda_2)$. Since matrix P in (16) is row-stochastic, column-stochastic and irreducible, we can guarantee, by choosing the gain $\zeta \geq \frac{4}{\lambda_2 \delta T}$ for a pre-defined $0 < \delta < 1$, that observer (15) converges asymptotically to a consensus, i.e., $\eta(kT + \delta T) \to a_o \mathbf{1}$, where $\mathbf{1} \in \Re^n$ is the vector of 1s and $a_o = \mathbf{1}^T \eta(kT)/N$ [38, ch. 5.2]. Therefore, for all $i \in \{1, \ \cdots, \ N\}$ the distributed observer converges to the total charging needs of the network. ■

Remark 1: The total charging needs can also be estimated using the existing freeway traffic estimation algorithms, e.g., [42], [43] which estimate traffic flow on the road segments along the highway.

Based on the estimated total charging needs and via I2I communication, the service station computes the consensus value of the normalized queue, i.e., the value $\eta_0(k)$ in (9).

C. Distributed Algorithm of Scheduling at Service Stations

In this subsection we describe the distributed scheduling algorithm executed by each service stations along the highway based on the idea of the consensus algorithm [38, ch. 5.2.4]. Specifically, the goal is to design $p_i(k)$ in (2) for each service station so that

1) consensus is achieved as defined in (9),
2) all the EVs on the highway are charged at least at one of the service stations.

To this end, by the means of I2I communication (see figure 5), the service stations adjust $p_i(k)$ using the information on the length of queue of their neighboring stations according to

$$
\begin{aligned}
p_1^*(k) &= \frac{c_1\mu_1}{\alpha_1(k)}\left[\frac{\eta_0(k)}{3} - \frac{2x_1(k)}{3c_1\mu_1} + \frac{x_2(k)}{3c_2\mu_2} + \frac{g_1(k)}{c_1\mu_1}\right]\\
p_i^*(k) &= \frac{c_i\mu_i}{\alpha_i(k)}\left[\frac{\eta_0(k)}{3} + \frac{x_{i-1}(k)}{3c_{i-1}\mu_{i-1}} - \frac{x_i(k)}{c_i\mu_i} + \right.\\
&\qquad \left. + \frac{x_{i+1}(k)}{3c_{i+1}\mu_{i+1}} + \frac{g_i(k)}{c_i\mu_i}\right], i = 2, \cdots, (N-1)\\
p_N^*(k) &= \frac{c_N\mu_N}{\alpha_N(k)}\left[\frac{\eta_0(k)}{3} - \frac{2x_N(k)}{3c_N\mu_N} + \frac{x_{N-1}(k)}{3c_{N-1}\mu_{N-1}} + \frac{g_N(k)}{c_N\mu_N}\right]
\end{aligned} \tag{17}
$$

where $\alpha_i(k)$ denotes the number of EVs approaching the ith service station and $g_i(k)$ is given by the approximation in (13). Furthermore, the value $\eta_0(k) \geq 0$ in (17) which serves as a leader can be computed from the following equation

$$\sum_{j=1}^{N} p_j^*(k)\alpha_j(k) = \eta_i((k-1)T + \delta T) \tag{18}$$

which shows that the total number of EVs entering the service stations equals to the total charging needs.

The following theorem is the main result of the paper which shows that consensus (9) is guaranteed.

Theorem 1: Under queue dynamics (2) and the distributed scheduling algorithms (17), (18), all the charging stations are uniformly utilized and all EVs are charged at one of the service stations. Furthermore, the queues at all service stations are bounded if and only if the total charging needs is less than the total capacity of the service stations.

Proof: Let us define $z_i(k) \stackrel{\triangle}{=} x_i(k)/c_i\mu_i$. First we show that under (17), the consensus (9) is achieved. Substituting $\hat{g}_i(x_i)$ in (13) into (2) results in

$$z_i(k+1) = z_i(k) + \frac{f_i(k)}{c_i\mu_i} - \frac{c_i\mu_i z_i(k)}{1 + c_i\mu_i z_i(k)}. \tag{19}$$

Furthermore, the following closed-loop dynamics is obtained by substituting (17) into (19):

$$z(k+1) = Pz(k) + \frac{1}{3}[\eta_0(k)\mathbf{1} - z(k)], \tag{20}$$

where $z = [z_1 \ \ldots \ z_N]^T$. Since the matrix P in (16) is primitive and row stochastic, it follows from [38] that system (20) asymptotically reaches a consensus given by $z(k) \to \eta_0(k)\mathbf{1}$.

Next, we show further that by choosing $\eta_0(k)$ satisfying (18) then all EVs are charged at one of the service stations. When all EVs are charged, then the following constraint is satisfied

$$
\begin{aligned}
&\sum_{i=1}^{N-1} f_i\left(k - \sum_{j=1}^{N-1} d_{j,j+1}\right) + f_N(k) = \beta_N(k)\gamma_N(k)\\
&\quad + \sum_{i=1}^{N-1} \beta_i\left(k - \sum_{j=1}^{N-1} d_{j,j+1}\right)\gamma_i\left(k - \sum_{j=1}^{N-1} d_{j,j+1}\right).
\end{aligned} \tag{21}
$$

Under update law (17) and assuming $\gamma_i(k)$ is constant, at the steady state the constraint (21) can be written as

$$\sum_{i=1}^{N} f_i^*(k) = \sum_{i=1}^{N} \beta_i(k)\gamma_i(k)$$

where $f_i^*(k) = p_i^*(k)\alpha_i(k)$ and which is equal to (18). Hence, all the EVs are charged at least once at the service stations.

Finally, we prove that $z_i(k)$ is bounded if and only if the total charging needs is less than the total capacity of the service stations, which can be written as

$$\eta_i((k-1)T + \delta T) < \sum_{j=1}^{N} c_j\mu_j.$$

For showing the necessity ($\Longrightarrow$), first we observe that under update law (17), when the queues are bounded we have $f_i^*(k) < c_i\mu_i$ for all stations i. Hence, we know that

$$\sum_{j=i}^{N} f_j^*(k) < \sum_{j=1}^{N} c_j\mu_j.$$

Combining the above inequality with (18) yields

$$\sum_{j=i}^{N} f_j^*(k) = \eta_i((k-1)T + \delta T) < \sum_{j=1}^{N} c_j\mu_j.$$

We show the sufficiency ($\Longleftarrow$) by contradiction. Similar to the previous case, when the queues are not bounded, under the update law (17), we have $f_i^* \geq c_i\mu_i$ for all service stations i. Hence, we have

$$\sum_{j=i}^{N} f_j^*(k) \geq \sum_{j=1}^{N} c_j\mu_j.$$

Combining the above inequality with (18) results in

$$\sum_{j=i}^{N} f_j^*(k) = \eta_i((k-1)T + \delta T) \geq \sum_{j=1}^{N} c_j\mu_j$$

which completes the proof. ■

Note that from (17) and since $\frac{x_i(k)}{c_i\mu_i} \to \eta_0(k)$, we have

$$p_i^* \to \frac{(c_i\mu_i)^2\eta_0(k)}{\alpha_i(k)\,[1 + c_i\mu_i\eta_0(k)]}. \tag{22}$$

Furthermore, at the steady state the following equality holds

$$\sum_{j=1}^{N} \frac{(c_j\mu_j)^2\eta_0(k)}{1 + c_j\mu_j\eta_0(k)} = \eta_i((k-1)T + \delta T)$$

which shows the relation between the number of queues at the steady state and the EV's total charging needs. In the following we provide some remarks regarding the proposed distributed scheduling algorithm (17).

Remark 2: It is shown in the proof of the theorem that the constraint (21) is satisfied at the steady state. The constraint (21) can be rewritten as

$$f_N(k) = \beta_N(k)\gamma_N(k) + \sum_{i=1}^{N-1}\left[\beta_i\left(k-\sum_{j=1}^{N-1} d_{j,j+1}\right)\gamma_i\left(k-\sum_{j=1}^{N-1} d_{j,j+1}\right) - f_i\left(k-\sum_{j=1}^{N-1} d_{j,j+1}\right)\right].$$

The second term on the right hand side of the above equation represents the total number of EVs which are not charged at the previous $(N-1)$-service stations. Therefore, during the transient state, the last service station, i.e., station N projects its control input $p_N^*(k)$ computed from (17) into the above equality in order to guarantee that all the EVs get charged. Furthermore, in practice when $\beta_i(k) > 1$, the charging needs $\beta_i(k)\gamma_i(k)$ are updated accordingly by considering number of EVs that have been charged at some of the stations.

Remark 3: The value of $\eta_0(k)$ can be computed from (18) in a distributed fashion. First, we can write from (17) that $f_i^*(k) = p_i^*(k)\alpha_i(k) = a_i\eta_0(k) + b_i$ where b_i is computed from the states $\frac{x_i(k)}{c_i\mu_i}$ of the corresponding station and its neighbors. The equality (18) can then be written as

$$\left(\sum_{j=1}^{N} a_i\right)\eta_0(k) + \sum_{j=1}^{N} b_i = \eta_i((k-1)T + \delta T). \qquad (23)$$

Similar to (15), the service stations execute the following observer:

$$\dot{\overline{a}} = \zeta(-I+P)\overline{a}, \quad \dot{\overline{b}} = -\zeta(-I+P)\overline{b},$$

where $\overline{a} = [\overline{a}_1, \cdots, \overline{a}_N]^T$ (resp. $\overline{b} = [\overline{b}_1, \cdots, \overline{b}_N]^T$) and initial conditions $\overline{a}_i(0) = Na_i$ (resp. $\overline{b}_i(0) = Nb_i$). As shown in lemma 1, for all service stations i the states will converge to $\overline{a}_i \to \sum_{j=1}^{N} a_i$ and $\overline{b}_i \to \sum_{j=1}^{N} b_i$ respectively. Hence, each service station can compute $\eta_0(k)$ from (23) independently.

Remark 4: It is worth to note that under the update law (17) the condition the total charging needs is less than the total capacity of the service stations is necessary and sufficient for the queues to be bounded. This is due to the nature of the consensus based update law (17) which forces the service stations to equally utilize their capacity and thus the chargers at each station can be used close to its limit capacity.

Remark 5: Consensus is also achieved under (17) for different queue model and its stability can be analyzed using the corresponding stability condition similar to (3).

IV. Cooperative Control of EVs' Charging

In this section, we design a cooperative control $u_{\upsilon,i}(k)$ in (6) for each EV by solving distributively optimization problem (8). In other words, the objective is to achieve the optimally scheduled percentage of EVs to be charged at every service station given by the aforementioned scheduling algorithm (i.e., the solution to optimization problem (7)) while meeting its own state-of-charge constraint, as depicted in figure 5.

The EVs, through their local computation, solve optimization (8) in a distributed manner by negotiating among themselves based on their state of charge and by using *vehicle-to-vehicle* (V2V) communication. To this end, the communication network among the EVs moving toward the same service station needs to be connected (or facilitated by infrastructure via V2I and I2V communication). To be more precise, each EV is only required to exchange information locally with its surrounding EVs such that all EVs moving toward the same station are connected, i.e., the information from every EV can (indirectly) propagate to any other EVs. When approaching service station i, the EVs closest to the station will receive the information on $p_i^*(k), d_{i,i+1}$ and $\alpha_i(k)$ transmitted by the ith service station with the use of *infrastructure-to-vehicle* communication. Each EV belonging to the set $\mathcal{N}_i(k)$ with $|\mathcal{N}_i(k)| = \alpha_i(k)$ then utilize this information to compute independently the number of EVs entering service station i, i.e., $u_{total}^i(k)$ which is the solution to the following optimization

$$\underset{u_{total}^i(k)}{\operatorname{argmin}} \left(p_i^* - \frac{u_{total}^i(k)}{\alpha_i(k)}\right)^2.$$

Next, each EV check their residual battery level $e_{\upsilon,i}^-$ and set their control input $u_{\upsilon,i}(k) = 1$ if $e_{\upsilon,i}^- < d_{i,i+1}r_\upsilon^-$, i.e., when they cannot reach service station $(i+1)$ without getting charge at the ith station. Let s_i denote the number of EVs that cannot reach the next station without getting charge at station i. Note that the value s_i can be computed distributively via a consensus algorithm as described in the previous section. Without loss of generality, it is assumed that the drivers fully charge their batteries at the service station. The normalized required energy for the EVs until they are fully charged is then equal to

$$e_{\upsilon,i}^r = 1 - \frac{e_{\upsilon,i}^-}{e_{\upsilon,max}}.$$

The rest $\alpha_i(k) - s_i$ EVs then exchange their $e_{\upsilon,i}^r$ values with their neighbors and sort them in an ascending order distributively, for example using the method proposed in [44] and [45]. As a final step, the EVs with $(u_{total}^i(k) - s_i)$th largest value of $e_{\upsilon,i}^r$ set their input $u_{\upsilon,i}(k) = 1$. The reason why the EVs do not exchange their current energy level $e_{\upsilon,i}^-$ is because the information may reveal the charging habit of the owners. Moreover, in combination with the information on battery capacity, the average speed of the EVs on the highway and by assuming that the owners fully charged their batteries before departing to work, private information such as the residential location of the EV's owner may also be estimated and exposed. On the other hand, exchanging the values of the required energy $e_{\upsilon,i}^r$ provides a better way to preserve or hide this private information since the drivers do not necessarily fully charge their batteries at the service station. The pseudo-code of the algorithm is given in algorithm 1. Note that the decision whether an EV should

Algorithm 1 Distributed algorithm to solve optimization (8)

Require: $p_i^*, \alpha_i(k), d_{i,i+1}$ broadcasted by the service stations, a strongly connected communication topology between the EVs.

for $k = 1, 2, \ldots, T_{final}$ **do**
  for $i = 1, \ldots, N$ **do**
    set $\mathcal{S}_i = \{\}$
    for $v = 1, \ldots, \alpha_i(k)$ **do**
      $u^i_{total}(k) = \text{argmin } (p_i^* - \frac{u^i_{total}(k)}{\alpha_i(k)})^2$
      compute battery status $e^-_{v,i}$
      if $e^-_{v,i} < d_{i,i+1} r^-_v, i \in \mathcal{N}_i$ **then**
        set $u_{v,i}(k) = 1$
        $\mathcal{S}_i \leftarrow v$
      end if
    end for
    set $s_i = |\mathcal{S}_i|$
    compute normalized required energy

$$e^r_{v,i} = 1 - \frac{e^-_{v,i}}{e_{v,max}}$$

    rank $e^r_{v,i}$ distributively in an ascending order for $v \notin \mathcal{S}_i$ using the method in [44] and [45]
    set $u_{v,i}(k) = 1$ for $(u^i_{total}(k) - s_i)$th largest $e^r_{v,i}$
  end for
end for

be charged at a given service station can be implemented, e.g., as a recommendation to the driver, in existing in-car navigation technologies or as an app. in a smart phone.

Remark 6: The number of EVs at the service station as a result of algorithm 1 may not always match the ones computed by distributed scheduling algorithm (17). This is either due to the uncertainty of the arrival/flow of EVs at the service station which influences the queue, or the first constraint in (8) captured by $\beta_i(k)$ in (17), or the stochastic nature of the driver's decision making. Such an uncertainty could be modeled as a (bounded) perturbation injected into dynamics (20). Note that since distributed scheduling algorithm (17) is performed in real time, the value $\eta_0(k)$ will be adjusted accordingly by taking into account the uncertainty of $x_i(k)$ at each station. If the total charging needs is less than the total charging capacity of the service stations and since the perturbation is bounded, the value $\eta_0(k)$ will remain bounded as shown in theorem 1. As a result, the states $\frac{x_i(k)}{c_i \mu_i}$ will attempt to track the value of $\eta_0(k)$, will reach consensus whenever $\eta_0(k)$ converges, and will stay bounded around $\eta_0(k)$ when $\eta_0(k)$ oscillates. Mathematically, the closed-loop dynamics (20) under bounded uncertainty $w(k) \in \mathbb{R}^N$ with $\|w(k)\|_\infty \leq \overline{w}$ can be written as

$$\begin{bmatrix} \eta_0(k+1) \\ z(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{3}\mathbf{1} & P - \frac{1}{3}I \end{bmatrix} \begin{bmatrix} \eta_0(k) \\ z(k) \end{bmatrix} + \begin{bmatrix} 0 \\ w(k) \end{bmatrix}. \quad (24)$$

It is shown in [38, Lemma 5.31] that the states $z(k)$ in (24) remains bounded given that $\eta_0(k)$ is bounded.

V. SIMULATIONS

In this section, we demonstrate and compare the performance of the proposed distributed scheduling using two different simulation examples on the Florida Turnpike, which is one of the highways in the United States.

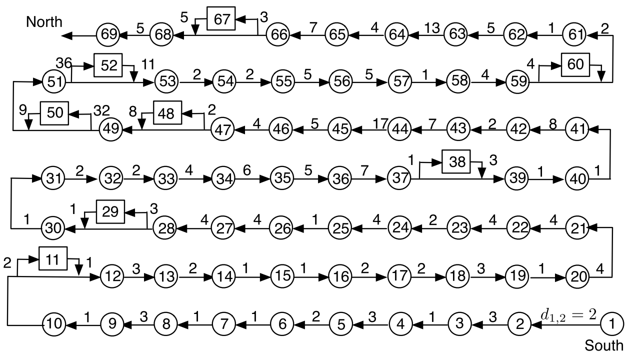


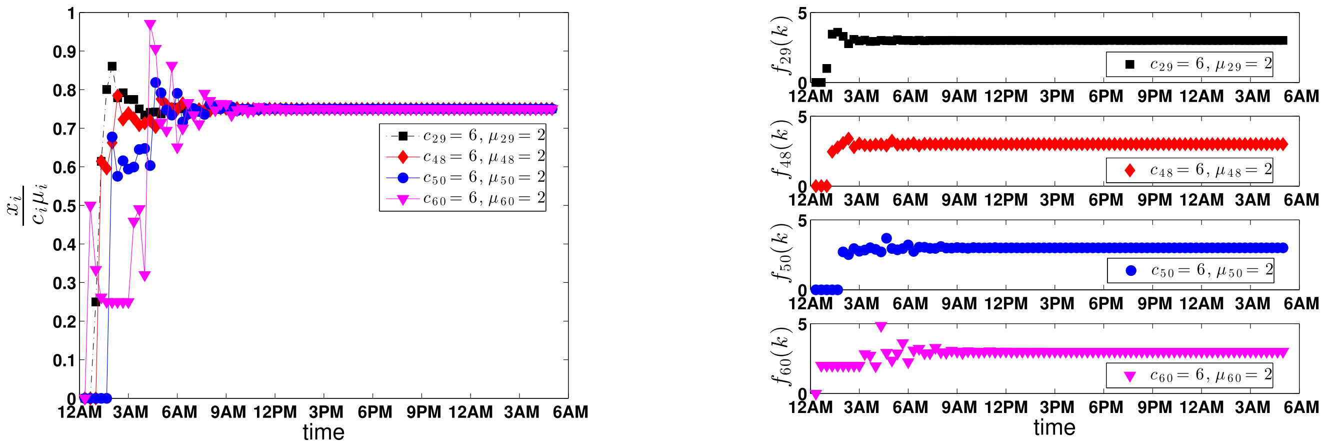
Fig. 7. The map of entrances/exits and service stations on Florida Turnpike. The circles represent entrances/exits and the rectangles represent service stations. The numbers above the arrows represent the distance (in minutes increments) between two consecutive nodes and the numbers in the circles denote the label of the entrances/exits and service stations.

Consider a Florida Turnpike consisting of 69 entrances/exits, i.e., $N = 69$ including 8 service stations as depicted in figure 7. The distances (in minutes increments) between the entrances/exits are summarized in figure 7 where it is assumed that the average velocity of the vehicles on the highway is equal to 60 miles/hour. For the sake of visualization and comparison, it is assumed that the EVs only need to be charged once along the highway, i.e., $\beta_i(k) = 1$ for all i. This is a realistic assumption since it is reported in [46] that EVs were charged 1.46 times per vehicle day driven on average. In addition, the simulation is performed for sampling time ΔT equal to 20 minutes for both the (physical) transportation and communication networks. We also assume that at the beginning of the day there are no EVs queueing at the service stations, i.e., $x_i(0) = 0$ for all i.

A. Algorithm Applied to Simulation Data

We first apply the proposed distributed algorithm to a constant traffic flow data where we set $\gamma_{14}(k) = 4$, $\gamma_{36}(k) = 6, \gamma_{54}(k) = 2$ and $\gamma_i(k) = 0$ for $i \neq \{14, 36, 54\}$. Furthermore, it is assumed that the six fast chargers are installed at each of the four service stations, namely $c_{29} = c_{48} = c_{50} = c_{60} = 6$ and $\mu_{29} = \mu_{48} = \mu_{50} = \mu_{60} = 2$ EVs/h. The simulation result is shown in figure 8a. As can be seen from the figure, all the four charging stations are uniformly utilized. Specifically, the consensus-based algorithm allocates equal number of EVs to each service station since all service stations have the same capacity as can be seen from figure 8b.

Next, we compare the performance of the proposed distributed scheduling algorithm for two different communication structures as illustrated in figure 9. To this end, we consider the case of four charging stations as before and assume that $\gamma_{14}(k) = 12$ and $\gamma_i(k) = 0$ for $i \neq 14$. Distributed algorithm corresponding to communication structure shown in figure 9a is given in (17). In addition, we modify (17) to incorporate the communication topology depicted in figure 9b where each charging station also receives information on the length of queues from the stations which are not its direct



(a) (b)

Fig. 8. Simulation results under the proposed consensus-based scheduling algorithm for constant traffic flow $\gamma_{14}(k) = 4, \gamma_{36}(k) = 6, \gamma_{54}(k) = 2$ with $c_{29} = c_{48} = c_{50} = c_{60} = 6$ and $\mu_{29} = \mu_{48} = \mu_{50} = \mu_{60} = 2$ EVs/h. (a) Utilization of service stations. (b) Number of EVs entering service stations.

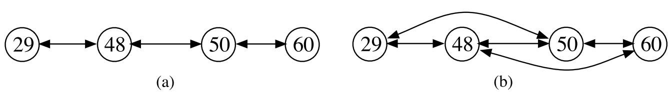


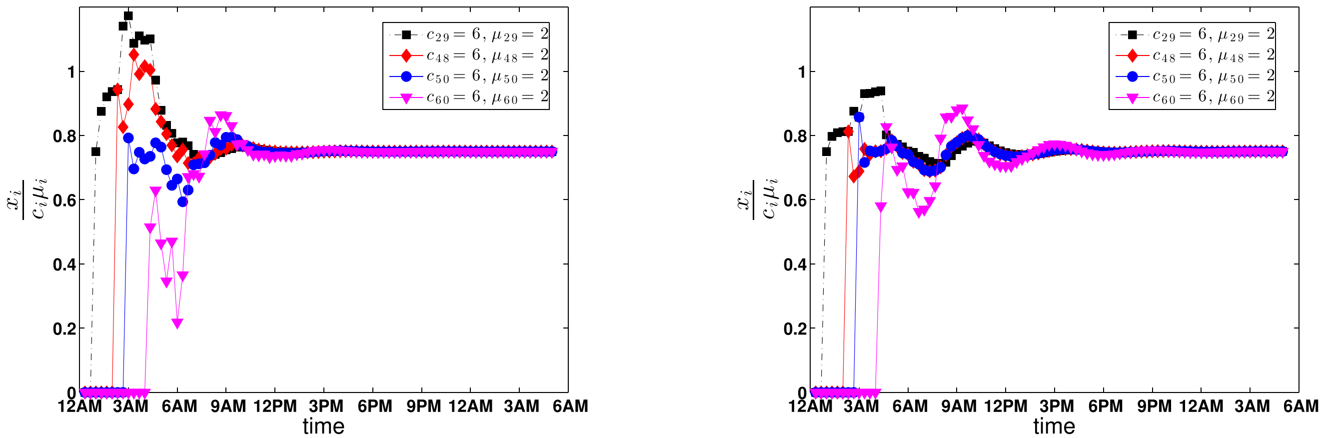
Fig. 9. Communication topologies of the service stations.

neighbors. The results are shown in figure 10. As can be observed from the figure, distributed algorithm based on both topologies result in the same utilization value at the steady state and the equal number of EVs are entering each service station. Furthermore, it can be seen that the topology given in figure 9b yields a better transient performance (i.e., the difference of utilization between the charging stations are smaller) in comparison to the communication topology in figure 9a. The result is intuitive since the charging stations in figure 9b receive more information than the ones in figure 9a.

B. Algorithm Applied to Real Florida Turnpike Data

Next, we utilize the real traffic data (per hour) obtained from Regional Integrated Transportation Information System (RITIS) database, namely the vehicles' net average flow at entrances/exits (nodes) 1 and 5 of the Florida turnpike, i.e., $\gamma_i(k) = 0$ for all $i \neq \{1, 5\}$ within 24 hours as depicted in figure 11. As can be observed, the traffic flow between 8-9 AM and between 6-8 PM are relatively higher than the rest of the day since during this two time frame, people are going to their work and going back home respectively. In addition, it is assumed that the EVs penetration rate is equal to 2% of the total vehicles' net average flow.

Currently, there are in total 12 units Tesla superchargers with charging time approximately 30 minutes installed along the highway, specifically 6 units each at the Fort Drum Service Plaza (node 50) and Turkey Lake Service Plaza (node 60) respectively [47]. Hence, we set $c_{50}, = c_{60} = 6$ and $\mu_{50} = \mu_{60} = 2$ Evs/h and $c_i = 0, \mu_i = 0$ for $i \neq \{50, 60\}$. For this particular simulation, the proposed consensus-based distributed scheduling algorithm is compared with an alternative strategy without any coordination/cooperation between the service stations. We call this alternative strategy as *State-of-Charge (SoC)-based random strategy* which employs the information on the battery's SoC of the EVs when approaching the service station. In this strategy, we first assigned a SoC threshold for charging given by S_t. When an EV approaching the i-th service station, it will check whether its SoC is below the threshold S_t. If its SoC is below S_t, then the driver will recharge the battery at the i-th station, i.e., $p_i(k) = 1$. Otherwise, the percentage $p_i(k)$ is given by a uniformly distributed random number, i.e., the EVs randomly decide whether to charge at station i. For the simulation, we set the SoC threshold S_t equal to 30% of the maximum SoC. This strategy is consistent with the findings in [46] where it is reported that most drivers charge their EV's battery when it is nearly fully depleted. Hence, the SoC based random strategy can approximately model the charging behavior of the EV's driver in the absence of any coordination (thus it is decentralized in nature). Note that a strategy similar to SoC-based random strategy is also employed in [22] for evaluating their algorithm where it is assumed that the EV's driver will choose the closest station when the SoC is below 30%. The authors in [26] also use a similar strategy, namely EVs will charge at the last station where they are able to reach, to compare and evaluate their charging algorithm. We perform monte carlo simulation (1000 trials with random initial SoC) for the SoC-based random strategy whose (average) result is shown in figure 12a. As can be seen, there is a large difference of utilization between the two service stations since no information of the current queue length of the neighboring station is used for deciding whether to charge at a specific service station. Next, we apply the proposed consensus-based scheduling algorithm whose result is illustrated in figure 12b. As can be observed, the consensus algorithm tries to allocate the EVs along the highway such that both service stations are uniformly utilized by using the information on the real time queue length at the neighboring stations. The number of EVs entering each service stations is illustrated in figure 14b. It is interesting to note that similar results on uniform utilization of charging stations are



(a) (b)

Fig. 10. Simulation results under the proposed consensus-based scheduling algorithm for different communication topologies and constant traffic flow $\gamma_{14}(k) = 12$ with $c_{29} = c_{48} = c_{50} = c_{60} = 6$ and $\mu_{29} = \mu_{48} = \mu_{50} = \mu_{60} = 2$ EVs/h. (a) Consensus-based algorithm for topology in figure 9a. (b) Consensus-based algorithm for topology in figure 9b.

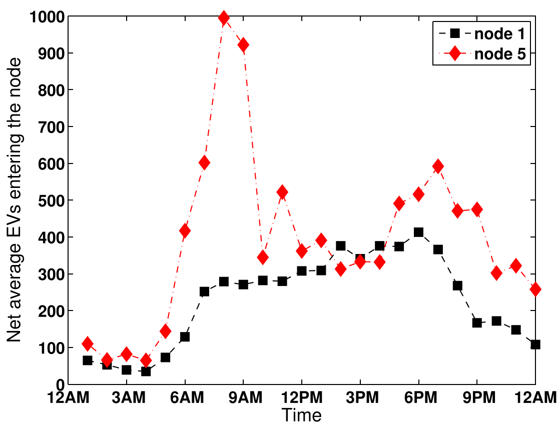


Fig. 11. Vehicles' net average flow entering nodes 1 and 5 per hour for 24 hours based on RITIS database. (a) Consensus-based algorithm for topology in figure 9a. (b) Consensus-based algorithm for topology in figure 9b.

also observed via simulations in [26] where the objective is to minimize the total travel time of individual EV and in [22] where the authors aimed at reducing the road congestion.

Finally, we perform the simulation when both service stations have different number of charges available. Specifically, we set $c_{50} = 5$ and $c_{60} = 7$ (note that the total charging capacity in the network remains the same). The simulation results by applying the SoC-based random strategy is depicted in figure 13a. The queues at both service stations vary significantly since most of the EVs decided to charge at the first station and no coordination is taken place. We improve the performance of the charging network by applying the consensus-based distributed scheduling algorithm whose result is shown in figure 13b. It can be observed that the proposed algorithm results in a uniform utilization of the service stations in spite of the heterogeneity of their capacity. Specifically, based on the information of the neighboring service stations' queues, the EVs are allocated "proportional" to the capacity of each service station as can be seen from figure 14b where the service station with more capacity takes more EVs to be charged.

TABLE II

ROOT MEAN SQUARE (rms) OF THE DIFFERENCE BETWEEN BOTH SERVICE STATIONS' UTILIZATION

Condition	SoC based strategy	proposed strategy
$c_{50} = c_{60} = 6$	1.21	0.29
$c_{50} = 5, c_{60} = 7$	3.53	0.21

TABLE III

COMPARISON OF TOTAL WAITING TIME T_i (IN MINUTES)

Condition	Node	SoC based strategy	proposed strategy
$c_{50} = c_{60} = 6$	50	44	22
	60	22	24
$c_{50} = 5, c_{60} = 7$	50	94	23
	60	18	21

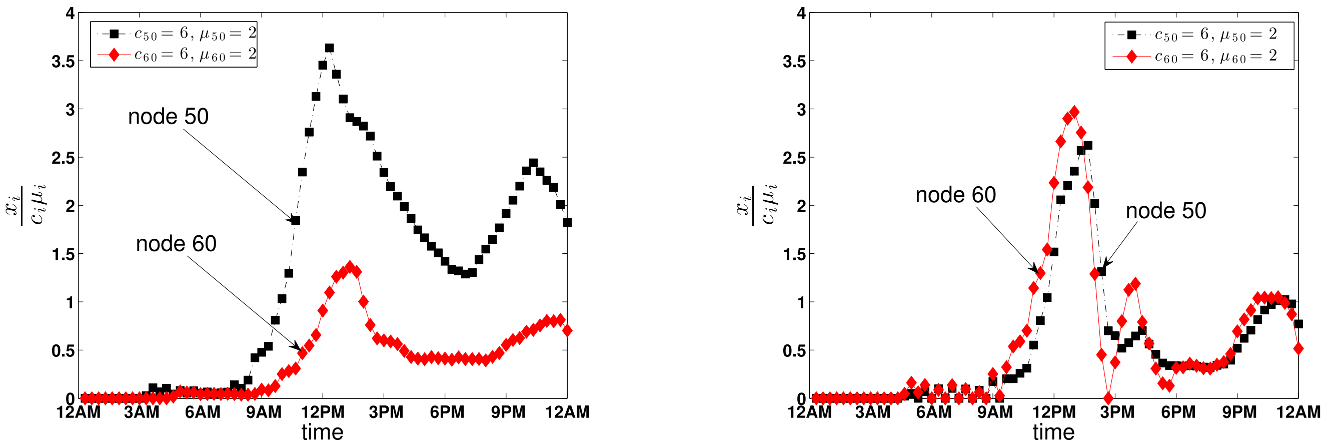
In order to further evaluate the performance improvement of the proposed strategy we also compute the root mean square of the difference between both service stations' utilization given by (setting $T_{final} = 72$)

$$rms = \sqrt{\frac{1}{72}\sum_{k=1}^{72}\left(\frac{x_{50}(k)}{c_{50}\mu_{50}} - \frac{x_{60}(k)}{c_{60}\mu_{60}}\right)^2}$$

and the results are summarized in table II. As can be seen from the table, the proposed algorithm yields a lower value of root-mean-square compared to the SoC-based random strategy. Moreover, we also compare the total waiting time for the EVs at each station. From Little's law formula, the total waiting time at the ith station T_i can be computed as [48]:

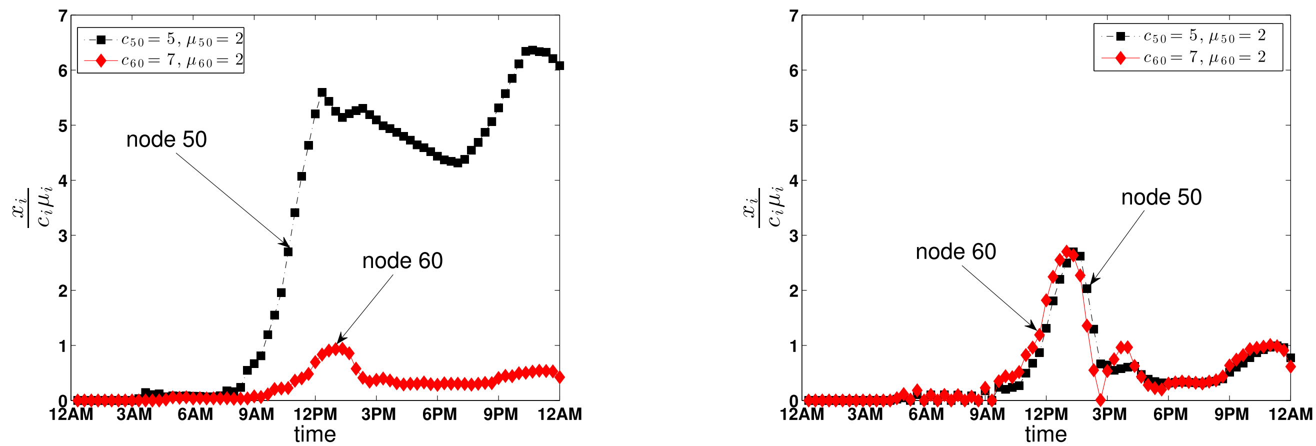
$$T_i = \Delta T\left(\frac{\sum x_i(k)}{\sum f_i(k)}\right).$$

Since additional time caused by the traffic congestion, i.e., driving time is assumed to be constant, the total waiting time is then equal to the increase of total travel time for the vehicles. It is interesting to note that, as discussed in [22], the uniform utilization of the service stations also results in the reduction of traffic congestions. The total waiting time for both the proposed strategy and SoC-based random strategy are summarized in table III. As can be seen from the table,



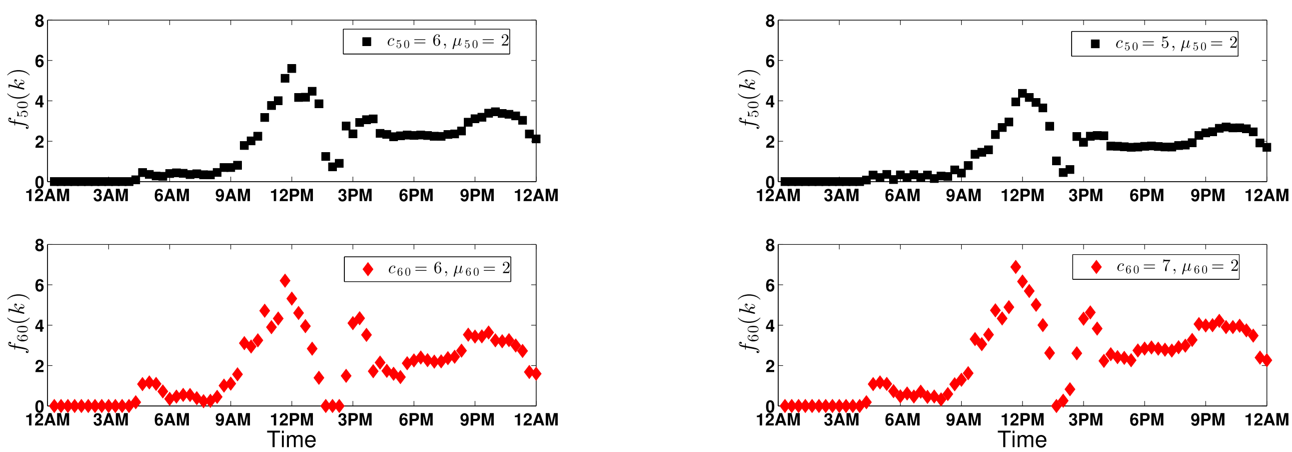
(a) (b)

Fig. 12. Utilization of service stations for real Florida turnpike data with $c_{50} = c_{60} = 6$ and $\mu_{50} = \mu_{60} = 2$ EVs/h. (a) SoC-based random charging strategy. (b) Consensus-based scheduling algorithm.



(a) (b)

Fig. 13. Utilization of service stations for real Florida turnpike data with $c_{50} = 5, c_{60} = 7$ and $\mu_{50} = \mu_{60} = 2$ EVs/h. (a) SoC-based random charging strategy. (b) Consensus-based scheduling algorithm.



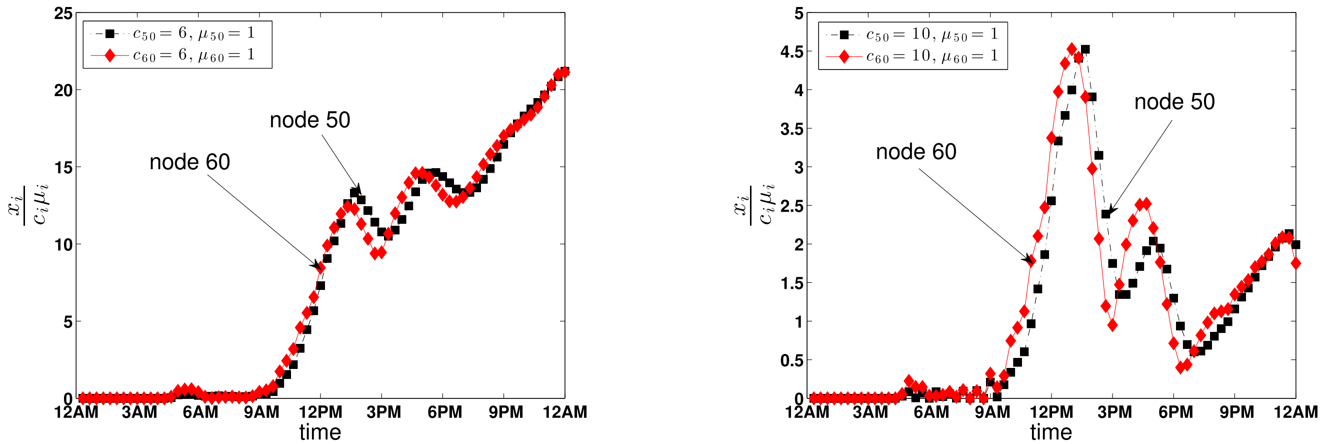
(a) (b)

Fig. 14. Number of EVs entering the service stations for real Florida turnpike data and by applying the proposed consensus-based scheduling algorithm.

the proposed strategy results in an (approximately) equal total waiting time for both service stations. Furthermore, the maximum total waiting time for the proposed strategy is lower than the random strategy.

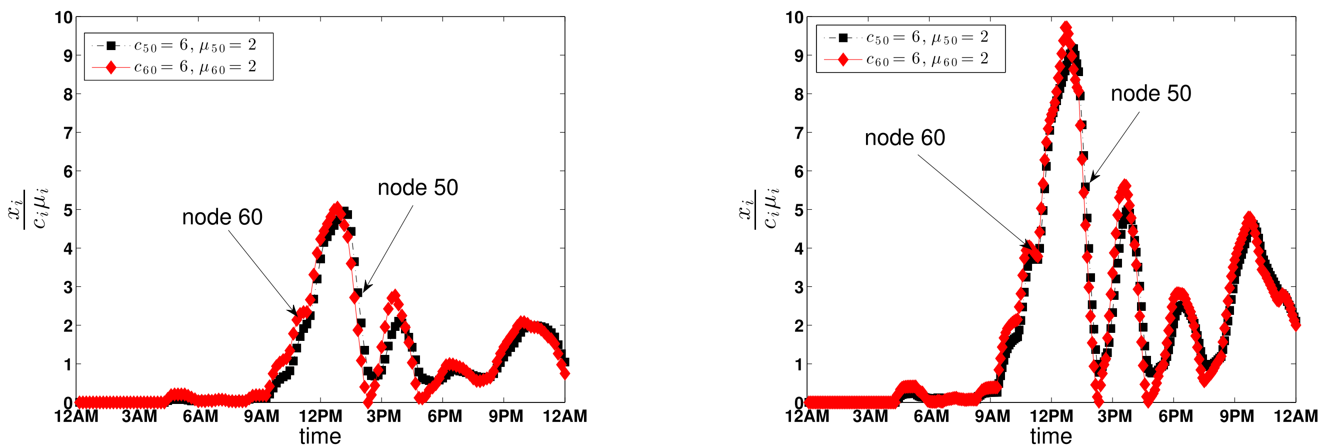
C. Evaluation of Proposed Algorithm for Different Parameters

In the following we evaluate the performance of the proposed algorithm for different parameters, namely different type of chargers and sampling time. First, we consider the case



(a) (b)

Fig. 15. Utilization of service stations for real Florida turnpike data with $\mu_{50} = \mu_{60} = 1$ EV/h. (a) $c_{50} = c_{60} = 6$. (b)$c_{50} = c_{60} = 10$.



(a) (b)

Fig. 16. Utilization of service stations for real Florida turnpike data for different sampling time. (a) ΔT = 10 minutes. (b) ΔT = 5 minutes.

where both service stations are equipped with chargers whose charging time is equal to 1 hour per EV while the number of chargers at each station $c_{50} = c_{60} = 6$. As can be seen from figure 13b, even though the service stations are uniformly utilized, the queues increase over time and become unbounded. This is because the number of EVs entering each station is larger than the capacity of the service station at most of the time. Next, we increase the number of charger at each station to $c_{50} = c_{60} = 10$. As a result, the queues at each service stations remain bounded as can be observed from figure 15b. Hence, for charger with charging time equal to 1 hour per EV, we require more chargers compared to the fast charging case in order to keep the queues at each service station bounded.

Finally, in order to study the sensitivity of the proposed algorithm with respect to the time k (or sampling time ΔT) we simulate the proposed algorithm for different sampling time (updated time), namely $\Delta T = 10$ minutes and $\Delta T = 5$ minutes respectively. It can be observed from figure 16 that the choice of ΔT impacts the utilization value $\frac{x_i}{c_i \mu_i}$ of the service stations. However, as can also be observed from the figure, the average length of queues at the service stations, i.e., x_i are about the same at a particular time of the day for different sampling time ΔT. In practice, the size of ΔT should be chosen depending on the traffic condition (e.g., volume of EVs on the highway). In addition, it is worth noting that in the paper we consider the worst case that communication is sampled at a slower rate (i.e., equal to the sampling rate of the transportation network). However, in practice the communication can be sampled at much faster rate depending on the traffic congestion or EVs speed, i.e., the consensus algorithm can be run asynchronously. This implies that the value $\eta_0(k)$ changes between the samples of the transportation/traffic model which makes the scheduling algorithm become more sensitive to the changes of traffic flow, i.e., the transient behavior.

VI. Conclusion

In this paper, we have developed a strategy consisting of a distributed scheduling algorithm and a cooperative control policy for individual EVs which optimize the operation of the overall charging network on a highway. First, a consensus-based distributed scheduling algorithm is presented which uses local information from the neighboring service stations and is designed so that all the charging stations are uniformly utilized. Next, we develop a negotiation strategy among the drivers by means of the V2V and V2I communications and based on their current battery level in order to meet the published scheduling level. It is confirmed from simulations that the proposed strategy improves the overall system performance compared to the SoC-based random strategy. It should be noted that by using graph theory (with nodes and edges represent entrances/exits/service stations and roads respectively), the highway transportation network studied in this paper can be extended to general road networks. Future research can be done to incorporate traffic congestions into the model, consideration of price-based strategy (game-theoretic approach), installment of new charging infrastructure including additional batteries for replacement to further reduce the total waiting time, control of traffic congestions by the adjustment of the maximum speed limit for the EVs, and performance analysis of the proposed distributed strategy under communication failures or delays.

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